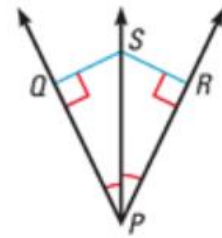


5.3 Use Angle Bisectors of Triangles

Remember that an *angle bisector* is a ray that divides an angle into two congruent adjacent angles. Remember also that the *distance from a point to a line* is the length of the perpendicular segment from the point to the line.

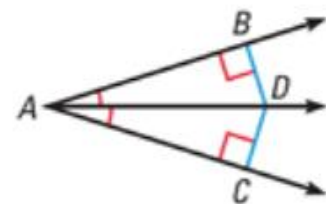
So, in the diagram, \overrightarrow{PS} is the bisector of $\angle QPR$ and the distance from S to \overrightarrow{PQ} is SQ , where $\overline{SQ} \perp \overrightarrow{PQ}$.



THEOREM 5.5 Angle Bisector Theorem

If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle.

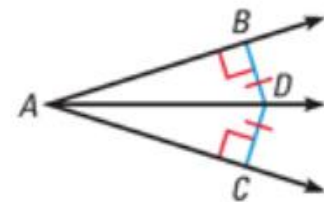
If \overrightarrow{AD} bisects $\angle BAC$ and $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$, then $DB = DC$.



THEOREM 5.6 Converse of the Angle Bisector Theorem

If a point is in the interior of an angle and is equidistant from the sides of the angle, then it lies on the bisector of the angle.

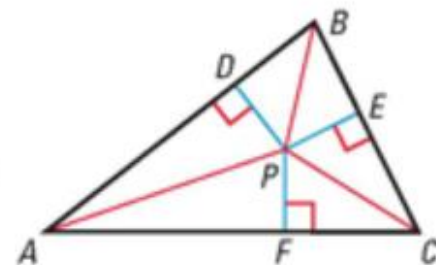
If $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$ and $DB = DC$, then \overrightarrow{AD} bisects $\angle BAC$.



THEOREM 5.7 Concurrency of Angle Bisectors of a Triangle

The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle.

If \overline{AP} , \overline{BP} , and \overline{CP} are angle bisectors of $\triangle ABC$, then $PD = PE = PF$.



The point of concurrency of the three angle bisectors of a triangle is called the **incenter** of the triangle. The incenter always lies inside the triangle.

Because the incenter P is equidistant from the three sides of the triangle, a circle drawn using P as the center and the distance to one side as the radius will just touch the other two sides. The circle is said to be *inscribed* within the triangle.

