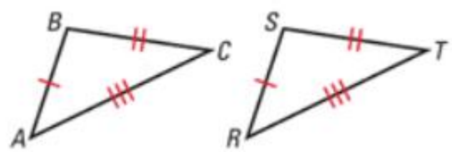
4.4/4.5 Prove Triangles Congruent by SSS, SAS and HL

POSTULATE 19 Side-Side-Side (SSS) Congruence Postulate

If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.

If Side $\overline{AB} \cong \overline{RS}$,
Side $\overline{BC} \cong \overline{ST}$, and
Side $\overline{CA} \cong \overline{TR}$,
then $\triangle ABC \cong \triangle RST$.

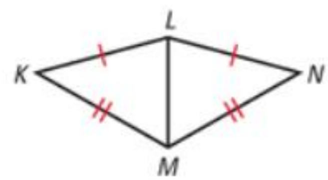


EXAMPLE 1 Use the SSS Congruence Postulate

Write a proof.

GIVEN ▶ $\overline{KL} \cong \overline{NL}$, $\overline{KM} \cong \overline{NM}$

PROVE ▶ $\triangle KLM \cong \triangle NLM$

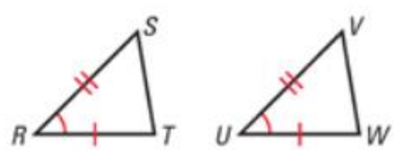


Proof It is given that $\overline{KL} \cong \overline{NL}$ and $\overline{KM} \cong \overline{NM}$. By the Reflexive Property, $\overline{LM} \cong \overline{LM}$. So, by the SSS Congruence Postulate, $\triangle KLM \cong \triangle NLM$.

POSTULATE 20 Side-Angle-Side (SAS) Congruence Postulate

If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

If Side $\overline{RS} \cong \overline{UV}$,
Angle $\angle R \cong \angle U$, and
Side $\overline{RT} \cong \overline{UW}$,
then $\triangle RST \cong \triangle UVW$.

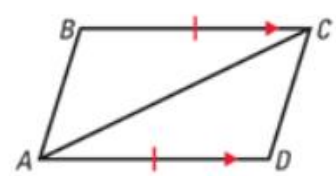


EXAMPLE 1 Use the SAS Congruence Postulate

Write a proof.

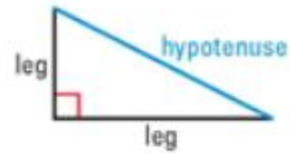
GIVEN ▶ $\overline{BC} \cong \overline{DA}$, $\overline{BC} \parallel \overline{AD}$

PROVE ▶ $\triangle ABC \cong \triangle CDA$



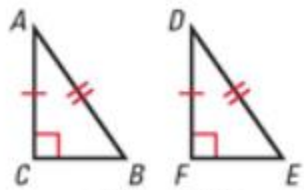
STATEMENTS	REASONS
S 1. $\overline{BC} \cong \overline{DA}$	1. Given
2. $\overline{BC} \parallel \overline{AD}$	2. Given
A 3. $\angle BCA \cong \angle DAC$	3. Alternate Interior Angles Theorem
S 4. $\overline{AC} \cong \overline{CA}$	4. Reflexive Property of Congruence
5. $\triangle ABC \cong \triangle CDA$	5. SAS Congruence Postulate

RIGHT TRIANGLES In a right triangle, the sides adjacent to the right angle are called the **legs**. The side opposite the right angle is called the **hypotenuse** of the right triangle.



THEOREM 4.5 Hypotenuse-Leg (HL) Congruence Theorem

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.



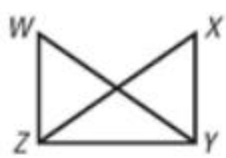
$\triangle ABC \cong \triangle DEF$

EXAMPLE 3 Use the Hypotenuse-Leg Congruence Theorem

Write a proof.

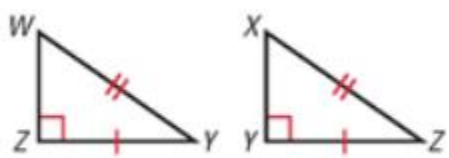
GIVEN ▶ $\overline{WY} \cong \overline{XZ}$, $\overline{WZ} \perp \overline{ZY}$, $\overline{XY} \perp \overline{ZY}$

PROVE ▶ $\triangle WYZ \cong \triangle XZY$



Solution

Redraw the triangles so they are side by side with corresponding parts in the same position. Mark the given information in the diagram.



	STATEMENTS	REASONS
H	1. $\overline{WY} \cong \overline{XZ}$	1. Given
	2. $\overline{WZ} \perp \overline{ZY}$, $\overline{XY} \perp \overline{ZY}$	2. Given
	3. $\angle Z$ and $\angle Y$ are right angles.	3. Definition of $\perp$ lines
	4. $\triangle WYZ$ and $\triangle XZY$ are right triangles.	4. Definition of a right triangle
L	5. $\overline{ZY} \cong \overline{YZ}$	5. Reflexive Property of Congruence
	6. $\triangle WYZ \cong \triangle XZY$	6. HL Congruence Theorem