

2.2 Analyze Conditional Statements

A **conditional statement** is a logical statement that has two parts, a *hypothesis* and a *conclusion*. When a conditional statement is written in **if-then form**, the “if” part contains the **hypothesis** and the “then” part contains the **conclusion**. Here is an example:

If **it is raining**, then **there are clouds in the sky**.



NEGATION The **negation** of a statement is the *opposite* of the original statement. Notice that Statement 2 is already negative, so its negation is positive.

Statement 1 The ball is red.

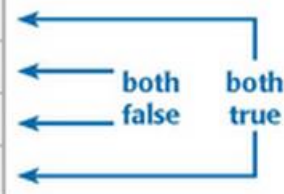
Statement 2 The cat is *not* black.

Negation 1 The ball is *not* red.

Negation 2 The cat is black.

RELATED CONDITIONALS To write the **converse** of a conditional statement, exchange the **hypothesis** and **conclusion**.

To write the **inverse** of a conditional statement, negate both the hypothesis and the conclusion. To write the **contrapositive**, first write the converse and then negate both the hypothesis and the conclusion.

Conditional statement If $m\angle A = 99^\circ$, then $\angle A$ is obtuse.	
Converse If $\angle A$ is obtuse, then $m\angle A = 99^\circ$.	
Inverse If $m\angle A \neq 99^\circ$, then $\angle A$ is not obtuse.	
Contrapositive If $\angle A$ is not obtuse, then $m\angle A \neq 99^\circ$.	

EXAMPLE 2 Write four related conditional statements

Write the if-then form, the converse, the inverse, and the contrapositive of the conditional statement “Guitar players are musicians.” Decide whether each statement is *true* or *false*.

Solution

If-then form If you are a guitar player, then you are a musician.
True, guitars players are musicians.

Converse If you are a musician, then you are a guitar player.
False, not all musicians play the guitar.

Inverse If you are not a guitar player, then you are not a musician.
False, even if you don’t play a guitar, you can still be a musician.

Contrapositive If you are not a musician, then you are not a guitar player. *True*, a person who is not a musician cannot be a guitar player.

DEFINITIONS You can write a definition as a conditional statement in if-then form or as its converse. Both the conditional statement and its converse are true. For example, consider the definition of *perpendicular lines*.

KEY CONCEPT

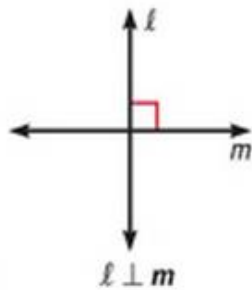
For Your Notebook

Perpendicular Lines

Definition If two lines intersect to form a right angle, then they are **perpendicular lines**.

The definition can also be written using the converse: If two lines are perpendicular lines, then they intersect to form a right angle.

You can write “line ℓ is perpendicular to line m ” as $\ell \perp m$.



BICONDITIONAL STATEMENTS When a conditional statement and its converse are both true, you can write them as a single *biconditional statement*. A **biconditional statement** is a statement that contains the phrase “if and only if.” Any valid definition can be written as a biconditional statement.

EXAMPLE 4 Write a biconditional

Write the definition of perpendicular lines as a biconditional.

Solution

Definition If **two lines intersect to form a right angle**, then **they are perpendicular**.

Converse If **two lines are perpendicular**, then **they intersect to form a right angle**.

Biconditional **Two lines are perpendicular** if and only if **they intersect to form a right angle**.