

11.2 Areas of Circles and Sectors

THEOREM 11.2 Area of a Circle

The area of a circle is π times the square of the radius.



Find the indicated measure.

a. Area

$$r = 2.5 \text{ cm}$$



b. Diameter

$$A = 113.1 \text{ cm}^2$$



Solution

$$\begin{aligned} \text{a. } A &= \pi r^2 && \text{Write formula for the area of a circle.} \\ &= \pi \cdot (2.5)^2 && \text{Substitute 2.5 for } r. \\ &= 6.25\pi && \text{Simplify.} \\ &\approx 19.63 && \text{Use a calculator.} \end{aligned}$$

▶ The area of $\odot A$ is about 19.63 square centimeters.

$$\begin{aligned} \text{b. } A &= \pi r^2 && \text{Write formula for the area of a circle.} \\ 113.1 &= \pi r^2 && \text{Substitute 113.1 for } A. \\ \frac{113.1}{\pi} &= r^2 && \text{Divide each side by } \pi. \\ 6 &\approx r && \text{Find the positive square root of each side.} \end{aligned}$$

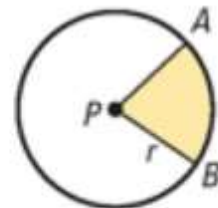
▶ The radius is about 6 cm, so the diameter is about 12 centimeters.

SECTORS A **sector of a circle** is the region bounded by two radii of the circle and their intercepted arc. In the diagram below, sector APB is bounded by \overline{AP} , \overline{BP} , and \widehat{AB} . Theorem 11.3 gives a method for finding the area of a sector.

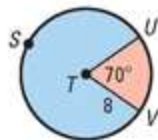
THEOREM 11.3 Area of a Sector

The ratio of the area of a sector of a circle to the area of the whole circle (πr^2) is equal to the ratio of the measure of the intercepted arc to 360° .

$$\frac{\text{Area of sector } APB}{\pi r^2} = \frac{m\widehat{AB}}{360^\circ}, \text{ or Area of sector } APB = \frac{m\widehat{AB}}{360^\circ} \cdot \pi r^2$$



Find the areas of the sectors formed by $\angle UTV$.



Solution

STEP 1 Find the measures of the minor and major arcs.

Because $m\angle UTV = 70^\circ$, $m\widehat{UV} = 70^\circ$ and $m\widehat{USV} = 360^\circ - 70^\circ = 290^\circ$.

STEP 2 Find the areas of the small and large sectors.

$$\text{Area of small sector} = \frac{m\widehat{UV}}{360^\circ} \cdot \pi r^2 \quad \text{Write formula for area of a sector.}$$

$$= \frac{70^\circ}{360^\circ} \cdot \pi \cdot 8^2 \quad \text{Substitute.}$$

$$\approx 39.10 \quad \text{Use a calculator.}$$

$$\text{Area of large sector} = \frac{m\widehat{USV}}{360^\circ} \cdot \pi r^2 \quad \text{Write formula for area of a sector.}$$

$$= \frac{290^\circ}{360^\circ} \cdot \pi \cdot 8^2 \quad \text{Substitute.}$$

$$\approx 161.97 \quad \text{Use a calculator.}$$

► The areas of the small and large sectors are about 39.10 square units and 161.97 square units, respectively.